Neutrino Quantum Kinetics

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Neutrinos in Supernovae

- Sufficiently massive stars fuse elements to iron
- Iron accumulates in core
- When core reaches Chandrasekhar limit, it collapses to a neutron star
- Gravitational binding energy released in neutrinos
- Neutrinos interact with matter above the core
- About 1% of neutrinos scatter far above the core, but since there are so many, this transfers enough energy to blow up the star

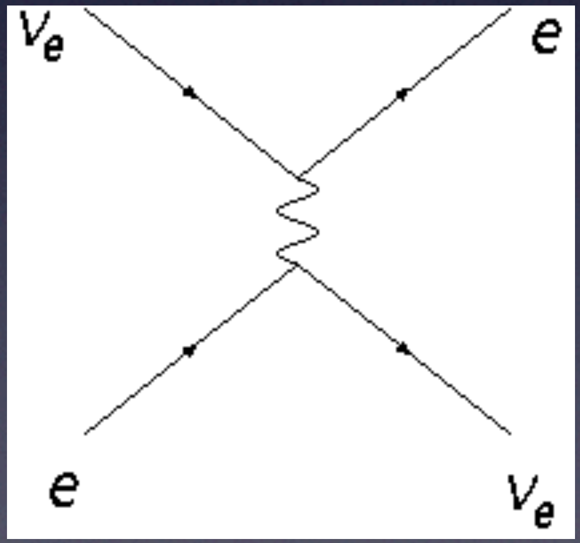
The story is incomplete:

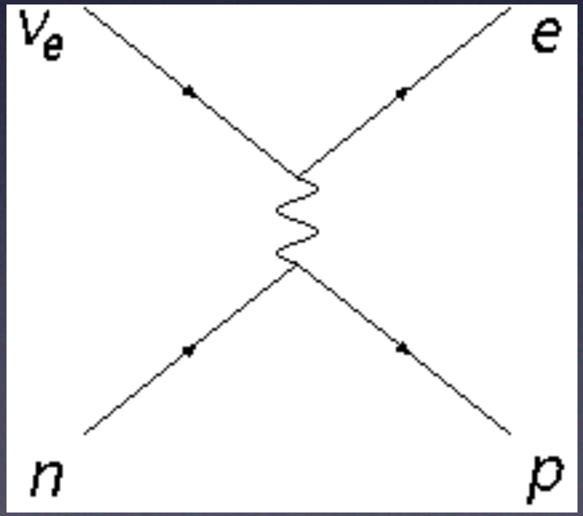
- We know that neutrinos undergo both inelastic scattering (crucial for the explosion mechanism, supernova nucleosynthesis, etc.) as well as quantum-mechanical flavor transformation
- No self-consistent approach currently exists that can treat both types of phenomena
- I will briefly discuss current approaches, then present a first-principles derivation of the correct description.

Current approaches, and their deficiencies

1. Supernova explosion models

- Use standard Boltzmann equation to describe neutrino scattering; ignore neutrino flavor evolution.
- However, electron neutrinos interact differently with matter than muon and tau neutrinos:





Thus neutrino flavor is important for energy transfer and nucleosynthesis. This issue is currently ignored in supernova models.

2. Supernova neutrino flavor transformation

Ignore inelastic or non-forward scattering, production and absorption of neutrinos.

Without collisions, we can solve a Schrödinger equation for a collection of single-particle wavefunctions:

$$i\frac{p_\mu}{E}\partial^\mu \left|\psi\right\rangle = H\left|\psi\right\rangle \qquad H = \frac{m^2}{2E} + H_{\nu e} + H_{\nu\nu}$$

$$\left|\psi\right\rangle = (\psi_e, \psi_\nu, \psi_\tau)$$

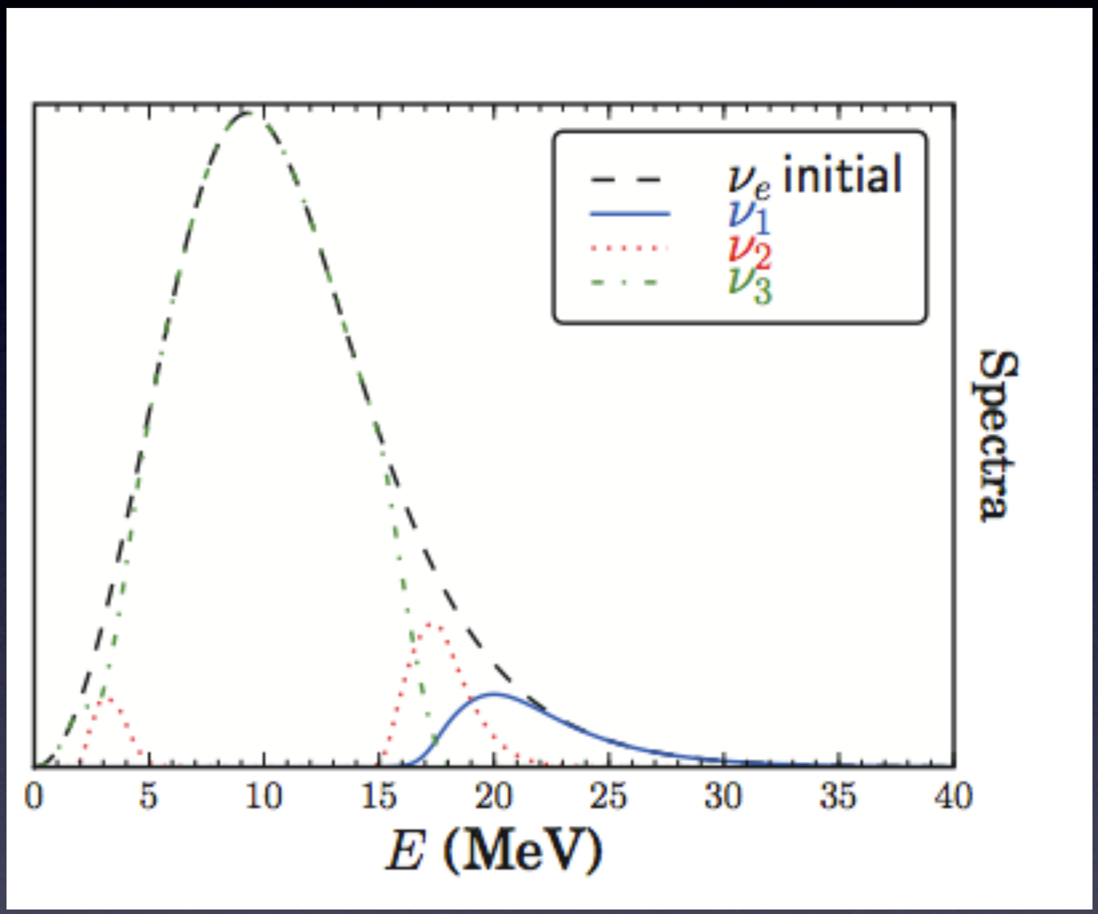
Vacuum Hamiltonian (mass)

Neutrino - matter interaction
(flavor-dependent index of refraction)

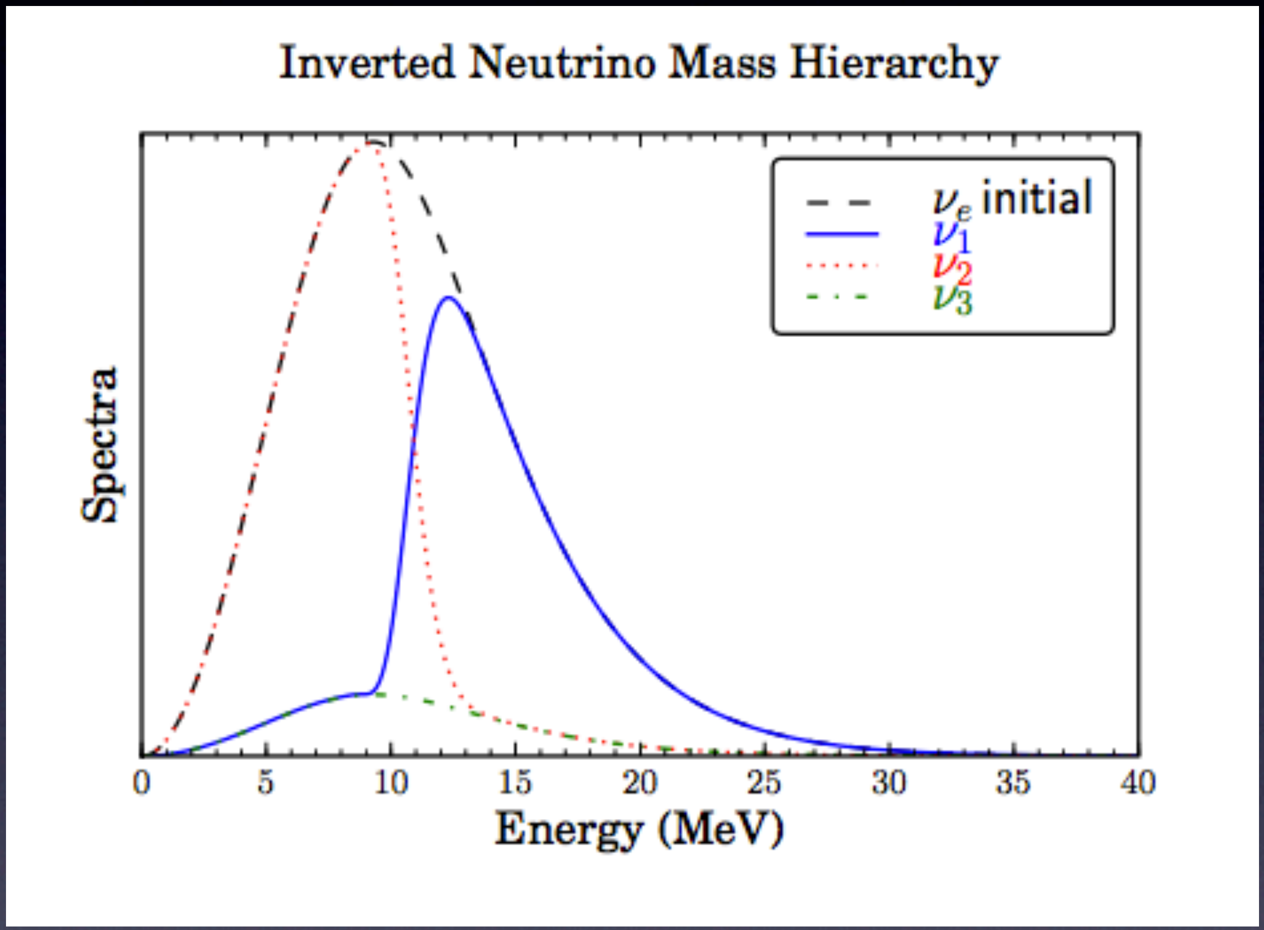
Nonlinear neutrino - neutrino interactions

Some results:

Collective flavor transformation over energy domains:



Normal Mass Hierarchy



Inverted Mass Hierarchy

Normal Mass Hierarchy:

ν_3

ν_2

ν_1

Inverted Mass Hierarchy:

ν_2

ν_1

ν_3

Flavor transformation critically depends on the nonlinear neutrino-neutrino interaction, requires the presence of a nonzero mass.

Note that the contribution to H from the mass is very small, so it may be surprising that flavor transformation occurs at all!

Current simulations typically show flavor transformation at a few hundred km radius, but this result may not be reliable.

Sensitive to fundamental neutrino parameters (mass hierarchy, mixing angles) as well as processes within supernova

Signatures of flavor transformation are detectable in the event of a Galactic supernova

Problems with this approach:

Scattering is assumed to be unimportant. This is motivated by two facts:

1. Few neutrinos scatter at >100 km

2. The spherically symmetric, collisionless models, show little or no flavor transformation at <100 km.

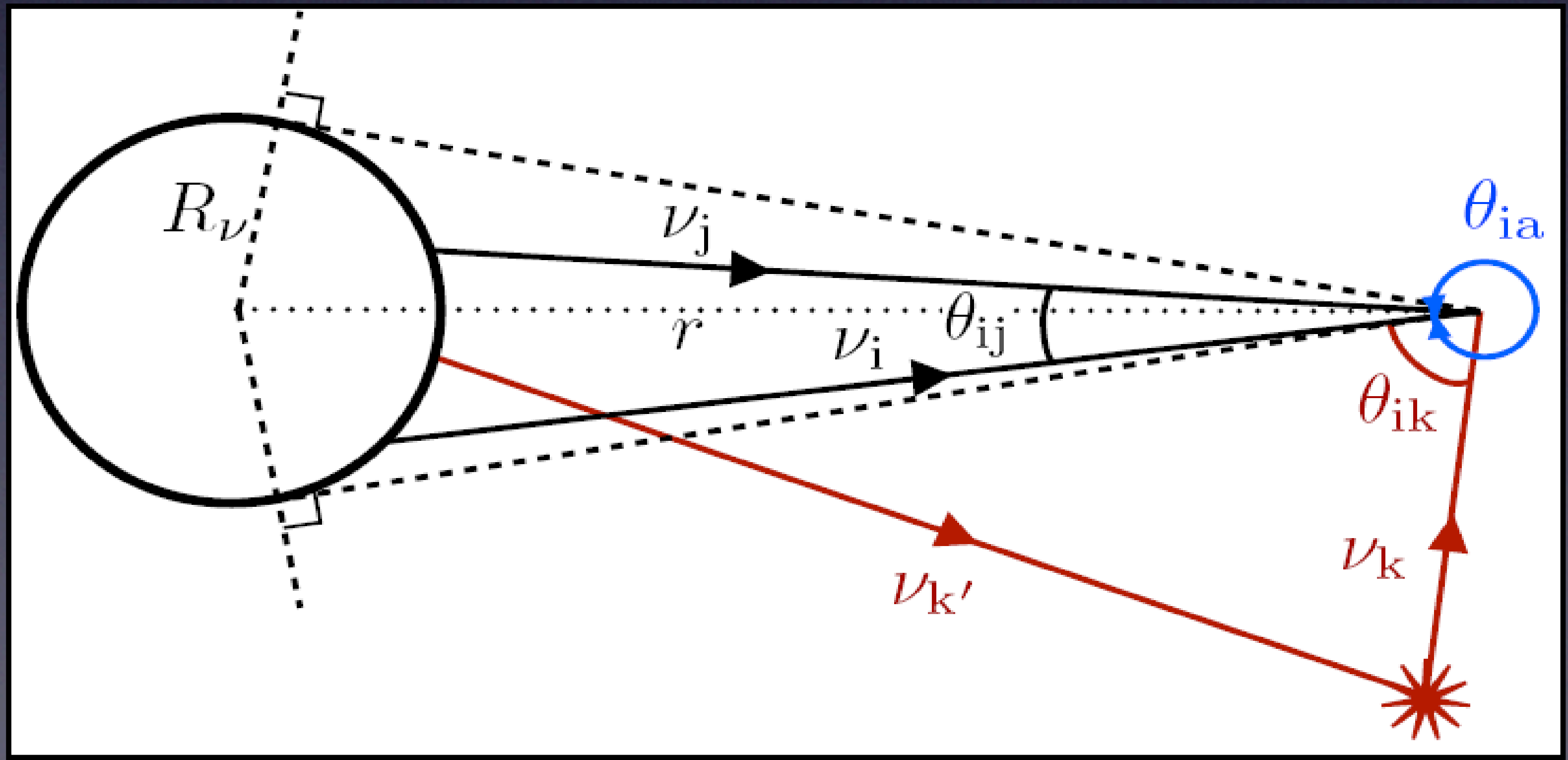
The hope is that we can use the Boltzmann equation to treat scattering close to the neutron star, and switch to coherent, collisionless approach further out.

Unfortunately, there is a problem with this:

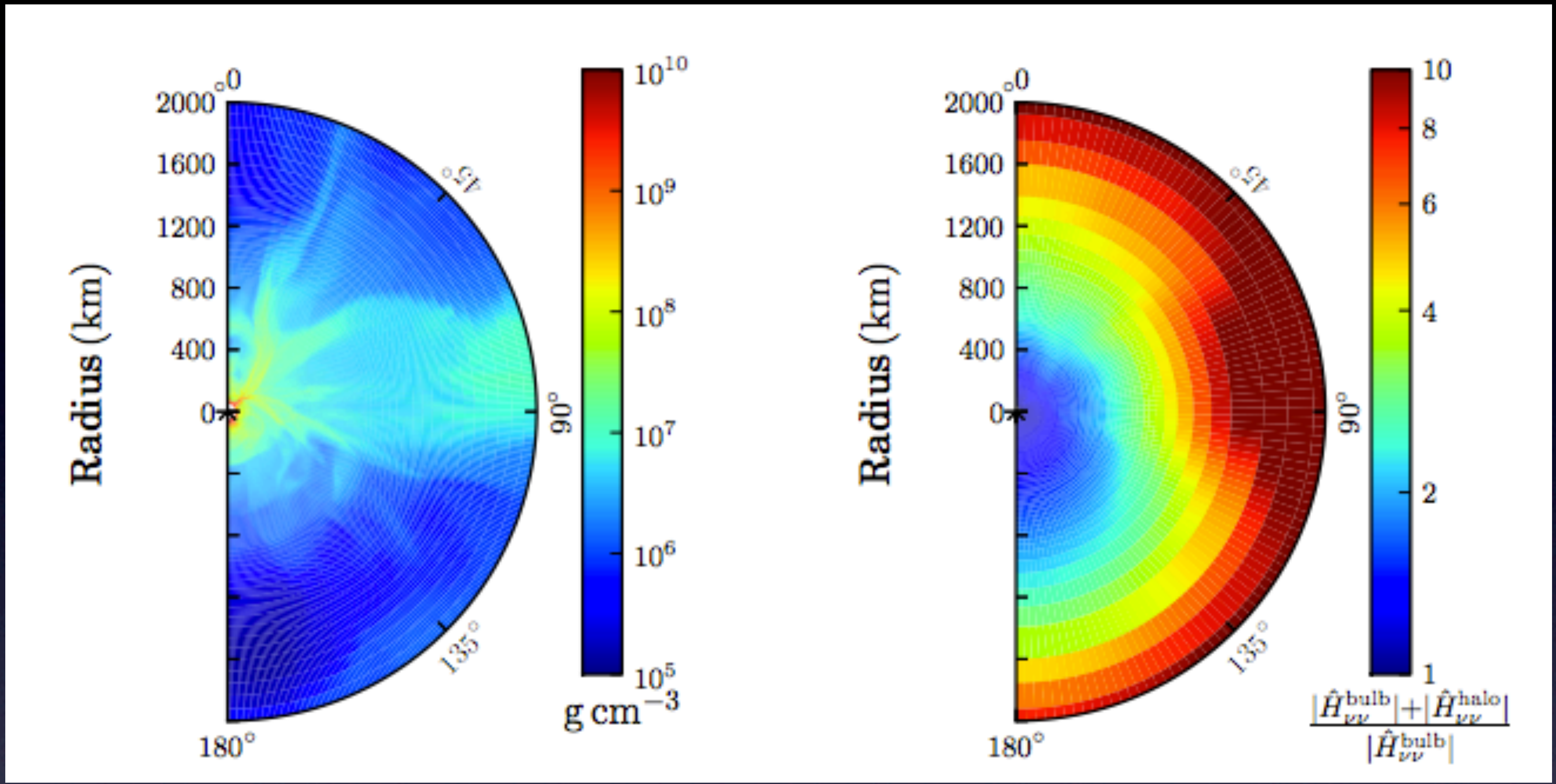
The Neutrino Halo

There are few scattered neutrinos above ~100km.

But, because of interaction geometry, scattered neutrinos ("the halo") can actually give the dominant contribution to neutrino-neutrino potential. (Cherry et al, 2012)



The Halo, Continued



Density profile of supernova

Relative contribution to $\nu\nu$ potential from scattered neutrinos

Note that simulations (which ignore the halo) typically show flavor transformation at a few hundred km - at this distance, the halo is not a correction, but the dominant contribution!

Cherry et al, arXiv:1302.1159. A simulation which includes a halo of scattered neutrinos, in a special case where this is easy to do. The halo is shown to have a large effect on flavor transformation.

Additional Assumptions & Missing Terms

Collisionless approach assumes no flavor transformation close in - this has not been proven in self-consistent way, since scattering is neglected, and the Hamiltonian is not correct for that regime.

Currently, a Hamiltonian of the following form is used:

$$H = \frac{m^2}{2E} + \Sigma \longleftarrow$$ Matter potential (from background + neutrinos)

It is assumed that $\Sigma << m$, so that terms of the form $\Sigma m / E$ and Σ^2 / E are neglected.

This assumption breaks down below ~100-200km! Some of the neglected terms can potentially lead to novel phenomena.

Our Approach

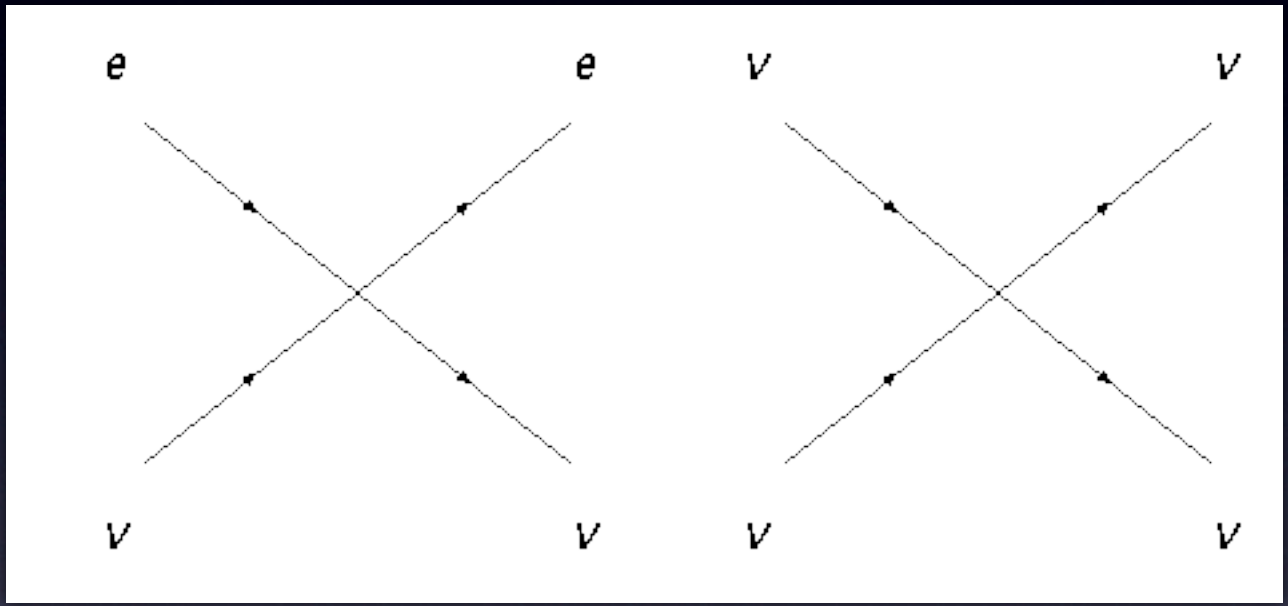
- We wish to obtain a correct generalization of the Boltzmann equations for flavored neutrinos, incorporating both inelastic scattering and quantum mechanical flavor evolution.
- Such analogues of the Boltzmann equation are known as Quantum Kinetic Equations (QKEs)

Previous Attempts at QKEs

- Early pioneers (Strack & Burroughs, Sigl & Raffelt) attempted to guess the form of the QKEs based on desired physical behavior - we can now say that their QKEs were not entirely correct.
- Berges (2005) showed that the Boltzmann equation could be derived directly from QFT
- More recently, QKEs for toy models of flavored scalar particles were derived from first principles (Cirigliano, 2010).
- We (Vlasenko, Fuller, Cirigliano, 2013, in prep.) now have the QKEs for Standard Model Majorana neutrinos from first principles. This is the first fully self-consistent, first-principles derivation of QKEs for flavored fermions.

Outline of Derivation

The model: 3 flavors of Majorana neutrinos, with Standard Model interactions (which reduce to 4-fermion vertices at low energies):



And similar interactions with nuclei & nucleons (these are not included in our first paper).

Need EOMs for particle densities: densities are part of the two-point function G(x,y) = <ψ(x)ψ(y)>, so calculate 2PI effective action and get EOMs for G

$$\Gamma_{2PI} = \Gamma_{2PI}^{\mathrm{vac}} + (2 - \mathrm{loop}) + (3 - \mathrm{loop}) \qquad \frac{\delta \Gamma^{2PI}}{\delta G\left(x,y\right)} = 0$$

Outline of Derivation, Continued

Incorporate finite neutrino densities, out of equilibrium, via the CTP formalism. Then, G decomposes as follows:

$$G\left(x,y\right)=\frac{1}{2}i\rho\left(x,y\right)\mathrm{sign}'\left(\mathrm{x}^0-\mathrm{y}^0\right)+\mathrm{F}\left(\mathrm{x},\mathrm{y}\right)$$

Propagator

Particle densities are in here somewhere

Take EOMs for F and Wigner transform:

$$F\left(x,p\right)=\int d^4re^{ip\cdot r}F\left(x-\frac{1}{2}r,x+\frac{1}{2}r\right)$$

Resulting equations for F:

$$\left(\not{p}+\frac{1}{2}i\not{\partial}-\tilde{\Sigma}-m\right)F\left(x,p\right)$$

$$=\frac{1}{2}i\left(\Pi^{+}\left(x,p\right)F^{-}\left(x,p\right)-\Pi^{-}\left(x,p\right)F^{+}\left(x,p\right)\right)$$

Now the fun begins:

F (statistical function) is a spin x flavor matrix
and thus has (4 x 4) x (3 x 3) components.

Eqn. for F relates some components, and gives EOMs for the rest.

We find the following dynamical quantities within F:

$f\,(x,p)$	3 x 3 Hermitean matrix	Neutrino density operator
$\bar{f}\,(x,p)$	3 x 3 Hermitean matrix	Antineutrino density op.
$\phi\,(x,p)$	3 x 3 complex matrix	Spin coherence (see below)

Final Form of the QKEs

$$iDf - [H, f] - U[\phi] = C\left[f, \bar{f}\right]$$

where

$$f = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix}$$

Diagonal terms = densities of neutrinos (by flavor)

Off-diagonal terms = coherence between flavors

$$iDf = i\frac{k_\mu}{E}\partial^\mu f + \frac{1}{2E}\left\{i\partial^i\Sigma^i, f\right\} + \ldots \quad H = \frac{m^\dagger m}{2E} + \Sigma^\kappa - \left[\Sigma^2\right]$$

$$U[\phi] = -\frac{1}{2E}\left(\Sigma^+ m^\dagger \phi^\dagger + \ldots + \ldots\right)$$

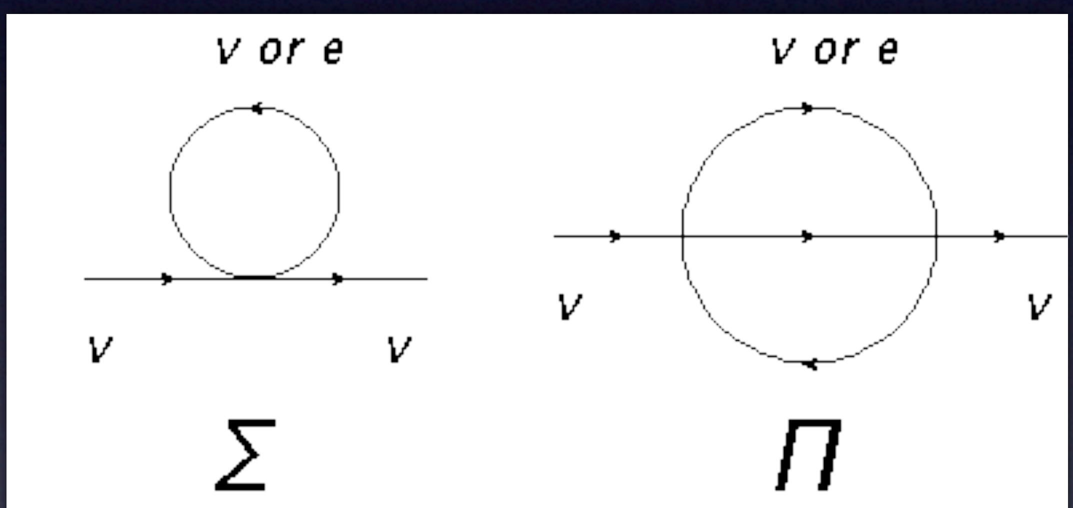
ϕ is a new dynamical quantity, which we interpret in a moment.

$$C = i\left\{\Pi^+, 1 - f\right\} - i\left\{\Pi^-, f\right\} + C[\phi]$$

This is the Boltzmann collision term, but with nontrivial flavor structure

Neutrino interactions:

Neutrino interactions are described by spin x flavor matrices Σ and Π, which are the self-energy diagrams:



Σ gives the forward scattering potential

$\Pi^{\pm}$ give the gain and loss potentials in the collision term

New Dynamical Quantities - Spin Coherence

We find a new dynamical quantity ϕ, which is coupled to the EOMs for the density operators via the $U(\phi)$ terms

Properties of the coupling terms:

- Proportional to neutrino vacuum mass multiplied by spacelike projection of matter potential - requires mass & anisotropy
- Conserve total neutrino plus antineutrino number for each momentum, but not the two separately.
- Thus, we interpret the effect of the coupling terms as coherent oscillations between neutrino and antineutrino states.

6 x 6 Formulation

The QKEs can be rewritten compactly as follows:

$$iD\mathcal{F} - [\mathcal{H}, \mathcal{F}] = i\mathcal{C}$$

where

$$\mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix} \qquad \mathcal{H} = \begin{pmatrix} H & H_\phi \\ H_\phi^\dagger & -\bar{H}^T \end{pmatrix}$$

$$H = \Sigma_k + \frac{m^\dagger m}{2E} + (\Sigma^2) \qquad H_\phi \propto \frac{m}{E}\left(\Sigma_x + i\Sigma_y\right)$$

H_ϕ mediates $\nu \leftrightarrow \bar{\nu}$ mixing

This term is small (down by $\frac{m}{E}$) but could potentially lead to large effects at resonance, especially w / nonlinearity!

Spin Coherence, Continued

- Mass and anisotropy requirement: neutrino-antineutrino oscillations are prohibited by lepton number conservation (broken by neutrino mass) and angular momentum conservation (broken by anisotropy)
- If neutrino oscillations can occur around 100km from neutrino sphere, they may be more likely to involve spin coherence than flavor, since $m\Sigma / E$ is larger than m^2 / E in this regime.
- Unlike flavor oscillations, spin oscillations are sensitive to the actual values of neutrino masses (not just mass differences)
- Sensitive to Dirac vs. Majorana nature of neutrinos. Dirac neutrinos can undergo spin oscillations into sterile states, while Majorana neutrinos can oscillate into (active) antineutrinos.

The Low-Density Coherent Limit

At low densities, $\Sigma << m$, and the collision term is much smaller still, so we can drop the collision terms, the Σ^2 terms and perhaps the $m\Sigma$ terms. Then, ϕ decouples, and we are left with:

$$i\frac{k_\mu}{E}\partial^\mu f - [H, f] = 0 \qquad H = \Sigma^\kappa + \frac{m^\dagger m}{2E}$$

$$i\frac{k_\mu}{E}\partial^\mu \bar{f} - [\bar{H}, \bar{f}] = 0 \qquad \bar{H} = \Sigma^\kappa - \frac{m^\dagger m}{2E}$$

This is a Schrödinger equation for neutrino and antineutrino density operators, equivalent to the equation for wave functions seen earlier.

The Boltzmann “Limit”

Requires several conditions:

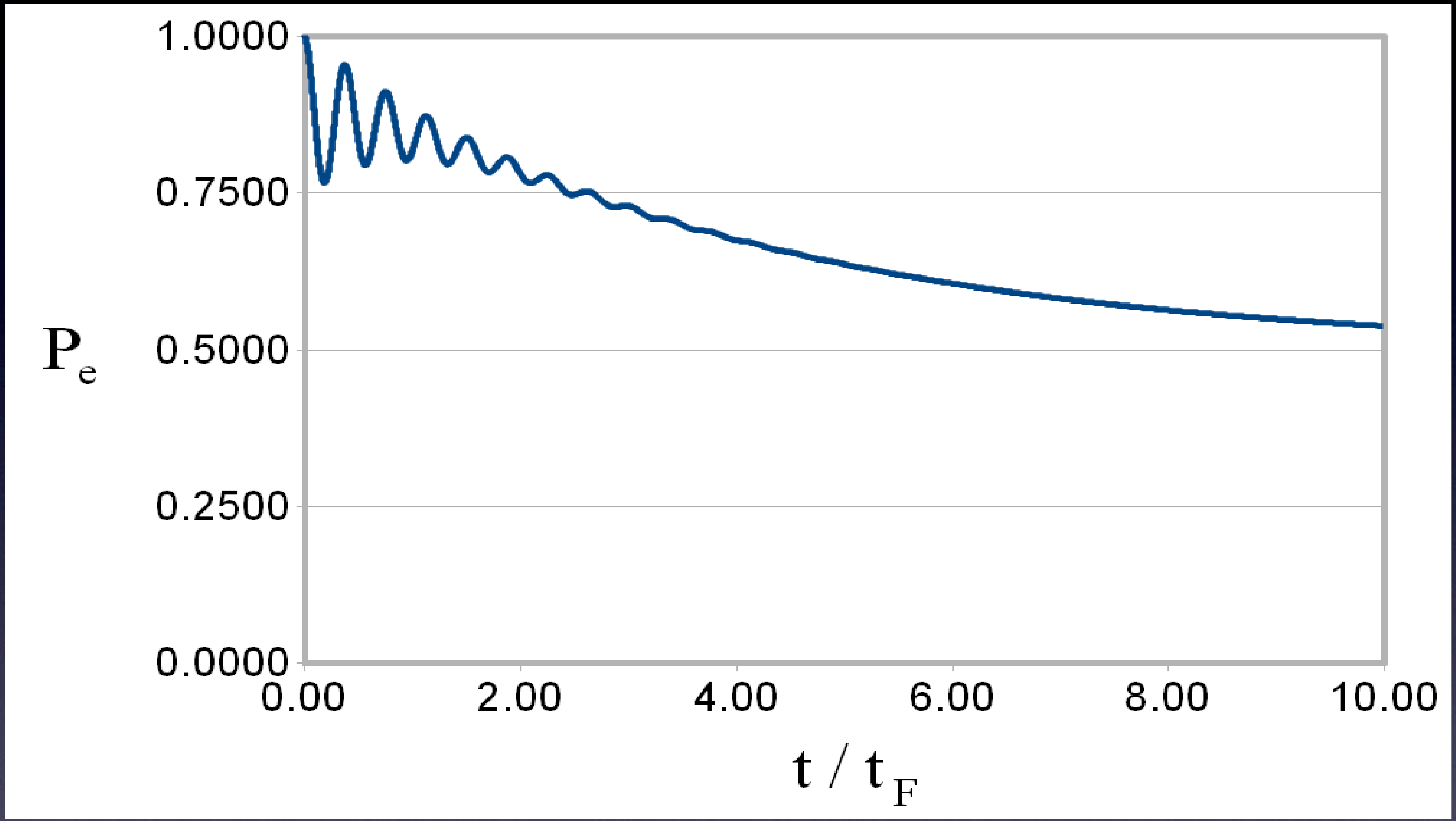
- The mass term is small enough to discard (this can happen if the matter potential is very large, but we have to be careful due to nonlinear nature of the eqns.)
- There is no coherence, so that [f, H] = 0 and $\phi = 0$
- Then, f is diagonal in the Hamiltonian basis, which, neglecting mass, is same as flavor basis - everything commutes. Collision term reduces to Boltzmann form.

$$\frac{k_\mu}{E}\partial^\mu f_I = \Pi_I^+ (1 - f_I) - \Pi_I^- f_I$$

The functions Π are the usual Boltzmann gain-loss expressions, and depend on neutrino and matter densities.

Decoherence & Flavor Depolarization

Decoherence in a simple isotropic model:



Note rapid damping of oscillations. This happens only when a collision term is present, and corresponds to $[H, f] \rightarrow 0$

Decoherence, Continued

In the Hamiltonian basis, $[H, f] \rightarrow 0$ corresponds to:

$$\begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \rightarrow \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix}$$

This is "wavefunction collapse": density operator for ensemble of "uncollapsed", coherent states looks like the LHS, while an ensemble of "collapsed" post-measurement states looks like the RHS.

Mechanism: collision term mixes different energy and flavor states, averaging out the off-diagonal terms.

No information is actually lost, but it is distributed among (infinitely) many neutrino states.

Conclusion

Deriving kinetic equations from field theory leads to some interesting results:

- Boltzmann equation and coherent flavor evolution can be obtained as limits in certain conditions
- New term gives the possibility of coherent transformation between neutrinos and anti-neutrinos
- Collisional decoherence / “wave function collapse” emerge naturally, without being put in by hand
- It is likely that QKEs must be solved, in one way or another, for a full description of neutrino flavor evolution